

# Constraints on the mSUGRA parameter space from electroweak precision data\*

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## Abstract

We place constraints on the parameter space of the minimal supergravity (SUGRA) inspired supersymmetric (SUSY) extension of the standard model (SM), *i.e.* the mSUGRA model, by studying the loop-level contributions of SUSY particles to electroweak precision observables. In general the Higgs bosons and the superpartner particles of SUSY models contribute to electroweak observables through universal propagator corrections as well as process-specific vertex and box diagrams. However, due to the bound on the mass of the lightest chargino,  $m_{\tilde{\chi}_1^\pm} > 91$  GeV, we find that the process-dependent contributions to four-fermion amplitudes are negligibly small. Hence, the full analysis may be reduced to an analysis of the propagator corrections, and in some regions of parameter space the constraints from the  $b \rightarrow s\gamma$  process are quite important. The propagator corrections are dominated by the contributions of the scalar fermions, and we summarize the results in the Peskin-Takeuchi  $S$ - $T$  plane and the contributions to the  $W$ -boson mass,  $m_W$ . We then present the results in the mSUGRA  $m_0$ - $m_{1/2}$  plane and find that our analysis of the propagator corrections provides constraints in the small- $m_0$ -small- $m_{1/2}$  region, precisely the region of interest for collider phenomenology. In some regions of parameter space, especially for  $\mu < 0$  and large  $\tan\beta$ , the constrained region is enlarged considerably by including the process  $b \rightarrow s\gamma$ .

This is the report of the Electroweak Precision Working Subgroup of the SUGRA Working Group for the Physics at Run II – Supersymmetry/Higgs Workshop. As such, we forgo the usual introduction and defer to the larger working-group report.[1] Our task is to place constraints on the parameter space of the minimal supergravity (SUGRA) inspired supersymmetric (SUSY) extension of the standard model (SM), *i.e.* the mSUGRA model, by studying the loop-level contributions of the supersymmetric particles to electroweak precision observables. The work presented here is part of a larger collaborative effort, and results will be presented more completely elsewhere.[2]

The loop-level contributions of supersymmetric (SUSY) particles to electroweak observables have been extensively discussed in the literature.[3, 4, 5, 6] In particular, processes with four external light fermions have been studied including observables which are sensitive to the  $Zbb$  coupling. The branching fraction  $\text{Br}(B \rightarrow X_s\gamma)$  is sensitive to SUSY effects in some regions of parameter space.[7, 8] The relationship between  $m_W$  and  $m_Z$  will provide stronger constraints as the measurement of  $m_W$  improves.

The complete one-loop corrections to four-fermion amplitudes include the universal propagator corrections as well as the process-dependent vertex and box corrections. However, when the extra Higgs bosons and the superpartner particles become sufficiently massive, it is necessary to retain only the leading propagator corrections[9], and these contributions may be summarized in terms of the  $S$ ,  $T$  and  $U$  parameters of Peskin and Takeuchi[10] or some other triplet of parameters.[11] The recent bounds[12] on the mass of the lightest chargino,  $m_{\tilde{\chi}_1^\pm} > 91$  GeV, and on the mass

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of the lighter scalar-top quark,  $m_{\tilde{t}_1} > 80$  GeV, imply a sufficiently massive spectrum such that the process-dependent vertex and box contributions may be safely neglected. In the context of the mSUGRA model, the chargino mass bound alone is sufficient to reach this conclusion.

In our analysis we adopt, in the notation of Hagiwara *et al.*[13], a form factor,  $g_L^b$ , to describe corrections to the  $Zbb$  vertex as well as the  $S$  and  $T$  parameters which include corrections to the gauge-boson propagators. We find that it is more convenient to drop the  $U$  parameter in favor of the directly measured  $W$ -boson mass. We first obtain constraints from the electroweak data on the four parameters  $\Delta g_L^b$ ,  $\Delta S$ ,  $\Delta T$  and  $\Delta m_W$ , which measure deviations from their corresponding SM reference values calculated at  $m_t = 175$  GeV and  $m_H = 100$  GeV. We then calculate the contributions to these parameters and to the  $B \rightarrow X_s \gamma$  decay width from the superpartner and Higgs particles to obtain constraints on the mSUGRA parameters.

The electroweak data through 1998 including the LEP and SLC experiments as well as low-energy neutral-current experiments may be summarized as

$$\begin{aligned} \Delta S - 24.2\Delta g_L^b &= -0.114 \pm 0.14 \\ \Delta T - 42.9\Delta g_L^b &= -0.215 \pm 0.14 \end{aligned} \quad \rho_{\text{corr}} = 0.77, \quad (1)$$

where  $\rho_{\text{corr}}$  denotes the correlation between the two one-sigma errors. Because the correlation is strong we present our results in the  $\Delta S' - \Delta T'$  plane where  $\Delta S' = \Delta S - 24.2\Delta g_L^b$  and  $\Delta T' = \Delta T - 42.9\Delta g_L^b$ . Note that  $m_W$  is not correlated with  $\Delta S'$  and  $\Delta T'$ , and hence it may be treated separately. Averaging the LEP2 and Tevatron measurements of the  $W$ -boson mass,  $m_W = 80.375 \pm 0.064$  GeV. The deviation of the data from the SM reference value for the  $W$ -boson mass is

$$\Delta m_W = -0.027 \pm 0.064 \text{ GeV}. \quad (2)$$

For the measurement of the branching fraction for the process  $b \rightarrow s\gamma$  we use

$$\text{Br}(B \rightarrow X_s \gamma) = 3.11 \pm 0.80 \pm 0.72 \times 10^{-4}, \quad (3)$$

from the ALEPH[14] collaboration. Results from the more recent CLEO measurement[15] will be reported elsewhere[2].

The SUSY contributions to  $\Delta S'$ ,  $\Delta T'$  and  $\Delta m_W$  are dominated by the contributions of the sfermions. Hence, we begin with a discussion of the sfermion contributions. In Figure 1(a) and (b) the ‘ $\times$ ’ marks the location of the best

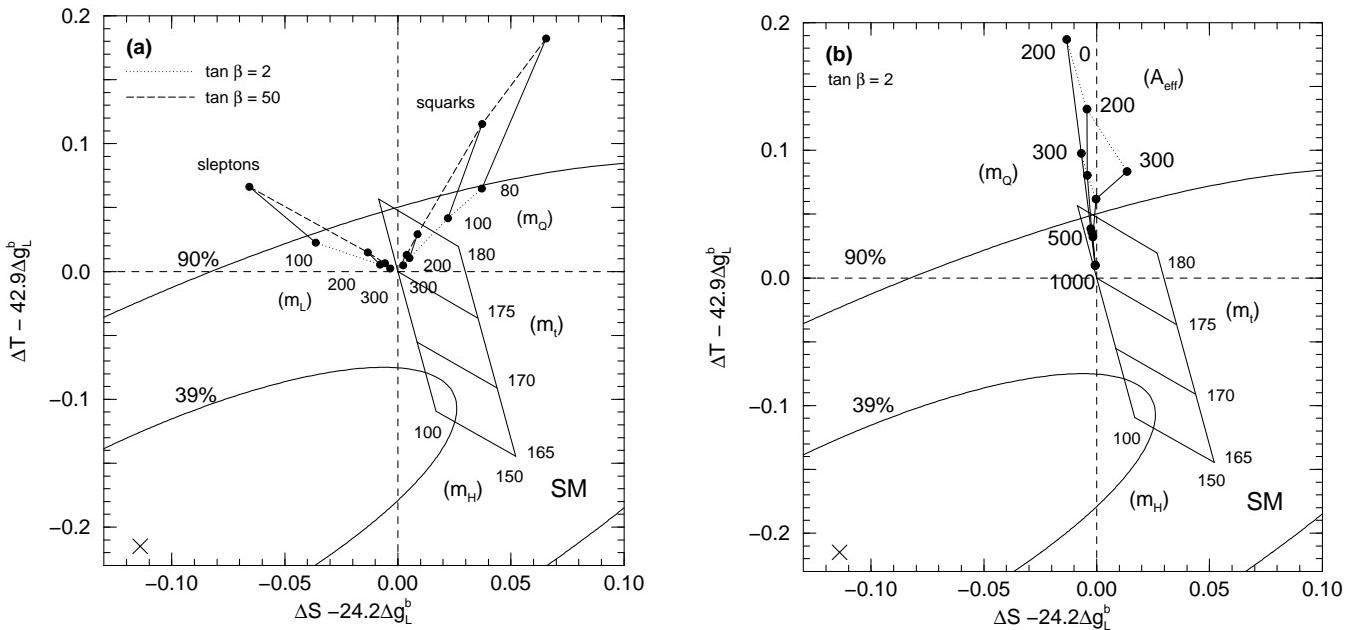


Figure 1: (a) shows the sfermion contributions for the first two families, and (b) shows the stop-sbottom contributions. Details are given in the text.

fit to the experimental data in the  $\Delta S' - \Delta T'$  plane, and the ellipses show the 39% (one-sigma) and 90% confidence-level (CL) contours as indicated. A grid has been included which shows the SM predictions for  $\Delta S'$  and  $\Delta T'$  as a

function of  $m_t$  and  $m_H$ . We choose the point where  $m_t = 175$  GeV and  $m_H = 100$  GeV as our reference point, *i.e.*  $\Delta S' = \Delta T' = 0$ , and the dashed-line axes are drawn through this point. The same point serves as the SUSY prediction in the limit of very large masses for the non-SM particles and when the lightest SUSY Higgs particle behaves like the SM Higgs boson.

Figure 1(a) includes the contributions of the sfermions of the first two generations with the squark and slepton contributions shown separately. The contribution of a sfermion loop to the  $S$  parameter is proportional to the hypercharge of the sfermion. Since  $Y = \frac{1}{6}$  for the squarks and  $Y = -\frac{1}{2}$  for the sleptons, we see that the squarks increase  $\Delta S'$  while the sleptons decrease  $\Delta S'$ . Dotted contours are used to show the case where  $\tan\beta = 2$  while dashed contours are used to show the  $\tan\beta = 50$  case. For the slepton contributions we show the cases where the explicit soft-SUSY-breaking slepton-doublet mass parameter has the nonzero values  $m_L = 100, 200$  and  $300$  GeV. Contours of equal  $m_L$  but varying  $\tan\beta$  are drawn using thin solid lines. Similarly we consider the squark contributions where the explicit soft-SUSY-breaking squark-doublet mass parameter has the values  $m_Q = 80, 100, 200$  and  $300$  GeV; contours of constant  $m_Q$  but varying  $\tan\beta$  are indicated by the thin solid lines. While the contributions to  $\Delta S'$  tend to cancel between the squark and sfermion sectors, the contributions to  $\Delta T'$  always add constructively, and for light sfermions lead to an unacceptably large deviation from the SM prediction and the experimental measurement of  $\Delta T'$ .

The large mass of the top quark leads to large left-right mixing of the top squarks, and to a lesser degree the mass of the bottom quark leads to left-right mixing of the bottom squarks. For this reason the third-family sfermions require a separate discussion, and we summarize the stop–sbottom contributions in Figure 1(b). In the mass matrix for the stop squarks it is the off-diagonal element  $-m_t A_{\text{eff}}^t$  where  $A_{\text{eff}}^t = A_t + \mu \cot\beta$  that determines the level of left-right mixing, while in the sbottom-squark mass matrix the off-diagonal element  $-m_b A_{\text{eff}}^b$  where  $A_{\text{eff}}^b = A_b + \mu \tan\beta$  determines the degree of mixing. We plot our results for  $A_{\text{eff}}^t = A_{\text{eff}}^b = A_{\text{eff}}$  showing contours of constant  $A_{\text{eff}}$  by the dashed lines and lines of constant  $m_Q$  by the dotted lines. In Figure 1(a) we saw that, with a value as small as  $m_Q = 80$  GeV, the contributions of the squarks of the first two generations to  $\Delta T'$  are still fairly small, while for the third family a value of  $m_Q = 300$  GeV already produces an unacceptable result for reasonable values of  $A_{\text{eff}}$ . It may be tempting to abandon universality of the soft-SUSY-breaking parameters and consider cases with a relatively small value of  $m_Q$  for the first two families and a much larger value to decouple the third family. While this is possible in principle, caution is required to avoid large flavor-changing neutral currents. In the context of the mSUGRA model we will, of course, use the soft-SUSY-breaking parameters which are obtained from the common mass parameters at the GUT scale. We also note that large values of  $A_{\text{eff}}$  tend to produce smaller  $\Delta T'$  but larger  $\Delta S'$ . We have shown only the case  $\tan\beta = 2$  since we find similar results for large  $\tan\beta$ .

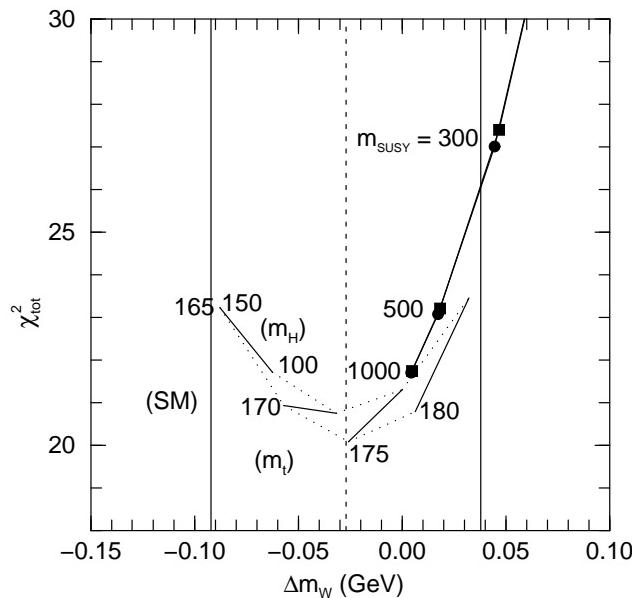


Figure 2: The sfermion contributions in the  $\chi_{\text{tot}}^2 - \Delta m_W$  plane where  $\chi_{\text{tot}}^2$  refers to the total  $\chi^2$  coming from the simultaneous fitting of  $\Delta S'$ ,  $\Delta T'$  and  $\Delta m_W$ .

Figure 2 shows the sfermion contributions to the  $W$ -boson mass. We include a grid that shows the SM prediction for  $\Delta m_W$  as a function of  $m_H$  and  $m_t$ . Along the upper dotted contour  $m_H = 100$  GeV, while the lower dotted contour corresponds to  $m_H = 150$  GeV. Points of equal  $m_t$  are connected by the solid line segments. The vertical dashed line represents the world average for the central value of the  $m_W$  measurement with the one-sigma errors represented by the vertical solid lines. For simplicity we set the explicit soft-SUSY-breaking squark-doublet, squark-singlet, slepton-doublet and slepton-singlet mass parameters to a common value,  $m_{\text{SUSY}}$ . We then plot the total chi-squared from the simultaneous fitting of  $\Delta S'$ ,  $\Delta T'$  and  $\Delta m_W$ , *i.e.*  $\chi^2_{\text{tot}}$ , versus  $\Delta m_W$  for  $\tan\beta = 2$  (represented by the squares) and  $\tan\beta = 50$  (represented by the circles). For  $m_{\text{SUSY}} = 1000$  GeV the  $\tan\beta = 2$  and  $\tan\beta = 50$  points are nearly indistinguishable. We note that the contributions of the SUSY particles always increase  $m_W$ . However, a value of  $m_{\text{SUSY}} = 300$  GeV leads to only a one-sigma discrepancy with the data. Hence, at the current time, the measurement of the  $W$ -boson mass provides only a minor constraint.

Although the Higgs bosons, the charginos and the neutralinos also contribute to  $\Delta S'$ ,  $\Delta T'$  and  $\Delta m_W$ , in the mSUGRA model the contributions are small compared to the sfermion contributions. Hence, even though we include these contributions in the numerical analysis, we do not show the Higgs-boson, chargino and neutralino figures that correspond to Figure 1 and Figure 2.

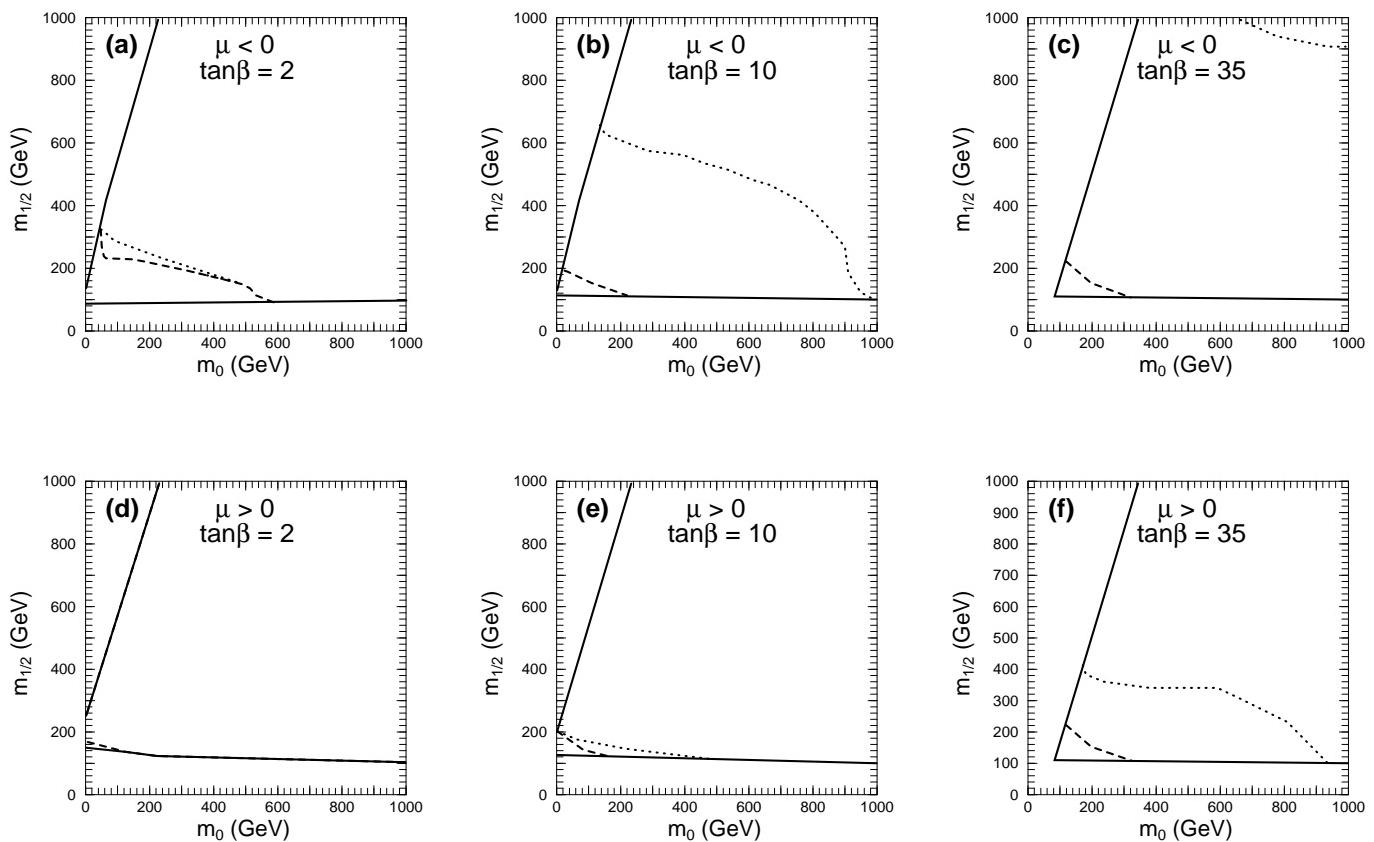


Figure 3: Favored regions in the mSUGRA  $m_0$ – $m_{1/2}$  plane lie in the region which is above and to the right of all drawn contours. Further explanation is provided in the text.

Next we discuss Figures 3(a)–(f). In each of these figures the values for  $\tan\beta$  and  $\text{sign}(\mu)$  are held to the constant values indicated. We allow  $A_0$  to vary in the range  $-500 \text{ GeV} < A_0 < 500 \text{ GeV}$ , and we scan the  $m_0$ – $m_{1/2}$  plane between 0 GeV and 1 TeV. For each point in the five-dimensional parameter space of unification-scale input parameters we employ the mSUGRA RGE portion of ISAJET[16] to determine the RGE evolution to the electroweak scale. We then verify whether that point is either excluded or allowed according to the following tests:

1. Verify that the obtained particle spectrum is physical, that the correct vacuum for electroweak symmetry breaking is obtained and that the lightest superpartner particle is a neutralino, *i.e.*  $\tilde{\chi}_1^0$ . This leads to a disallowed region in the upper left corner of each of the figures extending to the solid line with positive slope.

2. Verify that the chargino mass bound,  $m_{\tilde{\chi}_1^\pm} > 91$  GeV, is satisfied. We find that region below the horizontal solid line is excluded.
3. Calculate  $\Delta S'$ ,  $\Delta T'$  and  $\Delta m_W$  and check  $\chi^2_{\text{tot}}$ . Points which are disallowed at the 95% CL extend the disallowed region in the  $m_0$ - $m_{1/2}$  plane from the solid contour to the dashed contour.
4. Calculate the contribution to  $\text{Br}(B \rightarrow X_s \gamma)$ . Points which are disallowed at the 95% CL extend the disallowed region of the  $m_0$ - $m_{1/2}$  plane from the dashed contour up to the dotted contour.

The portion of the  $m_0$ - $m_{1/2}$  plane which is above and to the right of all the contours is deemed the ‘favored’ region for the mSUGRA model. The portion of the  $m_0$ - $m_{1/2}$  plane which is excluded by Test 2, the chargino mass bound, is significant. Once this has been taken into account, Test 3 excludes a corner of the remaining  $m_0$ - $m_{1/2}$  plane corresponding to small values of  $m_0$  and  $m_{1/2}$ . This region is fairly large in Figure 3(a) while it is barely observable in Figure 3(d). When  $\text{sign}(\mu) < 0$ , when  $\tan \beta$  is large, and especially when both of these conditions are true Test 4 excludes a significant region of the parameter space. In Figure 3(c) all but a tiny portion of the figure has been disallowed. Our excluded regions from Test 4 are larger than those of Ref. [8] due to a different treatment of strong corrections.

In conclusion, the direct constraints which come from the nonobservation of the lightest chargino at LEP2 have important consequences. First of all, the process dependent vertex and box corrections to four-fermion amplitudes become negligibly small, and as a result the analysis of electroweak data has been simplified and has become more transparent. After taking into account the chargino mass bound the  $Z$ -pole data, the low-energy neutral-current data and the measurement of the  $W$ -boson mass exclude only a small portion of the  $m_0$ - $m_{1/2}$  plane. However, this is still significant because the excluded region is where  $m_0$  and  $m_{1/2}$  are small, precisely the region of interest for collider studies, and especially relevant for the Tevatron. We find that the excluded region is largest for smaller  $\tan \beta$  with  $\text{sign}(\mu) < 0$ . For  $\text{sign}(\mu) < 0$  or  $\tan \beta$  large, a significant portion of the  $m_0$ - $m_{1/2}$  plane is excluded by the  $\text{Br}(B \rightarrow X_s \gamma)$  measurement, and the constraint becomes very severe when both of these conditions are met.

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